Chapter 3 – Mean Value Theorem WKST

Name: Key

Understanding IVT, MVT, and Rolle's Theorem

A Graphing Calculator is allowed for these problems.

For problems #1 and #2, find the value of c guaranteed by the Mean Value Theorem on the indicated interval such that $f'(c) = \dfrac{f(b) - f(a)}{b - a}$.

1. $f(x) = x^2$ on $[\overset{a}{1}, \overset{b}{3}]$

$f'(x) = 2x$

$f'(c) = 2c$

$f'(c) = \dfrac{f(3) - f(1)}{3 - 1}$

$2c = \dfrac{3^2 - 1^2}{3 - 1}$

$2c = \dfrac{9 - 1}{2}$

$2c = \dfrac{8}{2}$

$2c = 4$

$\boxed{c = 2}$

2. $f(x) = x^3 + 1$ on $[-1, 1]$

$f'(x) = 3x^2$

$f'(c) = 3c^2$

$f'(c) = \dfrac{f(1) - f(-1)}{1 - -1}$

$3c^2 = \dfrac{(1^3 + 1) - ((-1)^3 + 1)}{2}$

$3c^2 = \dfrac{2 - 0}{2}$

$3c^2 = 1$

$c^2 = \frac{1}{3}$

$c = \pm\sqrt{\frac{1}{3}} \to \boxed{c = \pm\frac{\sqrt{3}}{3}}$

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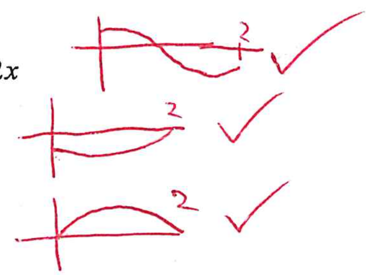
3. Which of the following satisfy the hypotheses of Mean Value Theorem on the interval $[0, 2]$?

I. $f(x) = \sin \pi x + \cos 2x$ ✓

II. $f(x) = \sqrt[3]{x - 2}$ ✓

III. $f(x) = |x^2 - 2x|$ ✓



(A) I only (B) II only (C) III only (D) I and II (E) I, II, and III

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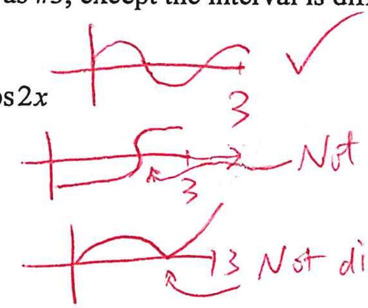
4. Which of the following satisfy the hypotheses of Mean Value Theorem on the interval $[0, 3]$?
Note: this is the same as #3, except the interval is different.

I. $f(x) = \sin \pi x + \cos 2x$ ✓

II. $f(x) = \sqrt[3]{x - 2}$ Not differentiable. $\sqrt[3]{\ }$ cube roots have vertical tangents

III. $f(x) = |x^2 - 2x|$ Not differentiable, sharp corner at $x = 2$.



(A) I only (B) II only (C) III only (D) I and II (E) I, II, and III

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5. Which of the following satisfy the hypotheses of Rolle's Theorem on the interval [0 , 2] ?

I. $f(x)=\frac{1}{|x-1|}$ Not Diff at $x=1$ since Vert Asymp.

II. $f(x)=|x-1|$ Not diff, sharp corner at $x=1$

III. $f(x)=x^2-2x$ ✓ YES, since $f(0)=f(2)$

IV. $f(x)=\sin 2x$ No since $f(0)\neq f(2)$

(A) I only (B) II only (C) III only (D) IV only (E) I and II

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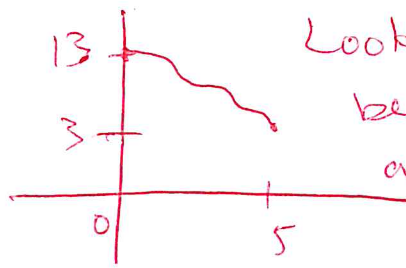
6. Let f be continuous for $0 \le x \le 5$ where (0 , 13) and (5 , 3) are the endpoints of f. The Intermediate Value Theorem guarantees which of the following?



Look for $f(c)$ between 3 and 13.

(A) $f(c)=2$ for some c such that $0<c<5$.

(B) $f'(c)=2$ for some c such that $0<c<5$.

(C) $f'(c)=0$ for some c such that $0<c<5$.

(D) $f(c)=4$ for some c such that $0<c<5$.

(E) $\lim_{x \to c} f(x)=f(c)$ for all values c on $0<c<5$.

This means continuous. Although this is true, the IVT doesn't guarantee this.

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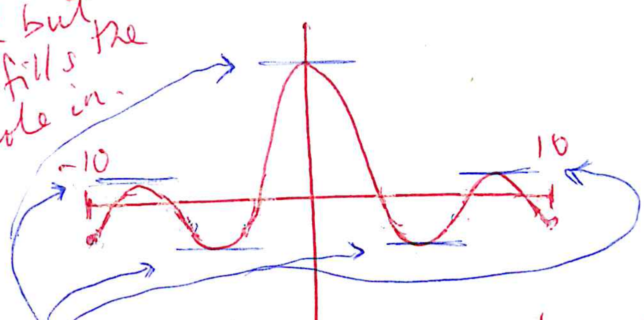
7. How many values of c are guaranteed by Rolle's Theorem for $f(x)$ below on the interval [-10 , 10] ?

undefined but this fills the hole in.

$$f(x)=\begin{cases} \frac{\sin x}{x}, & \text{for } x \neq 0 \\ 1, & \text{for } x=0 \end{cases}$$



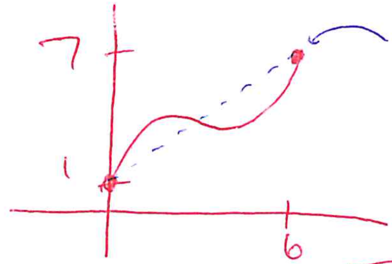
* Look how many times $f'(x)=0$ (horizontal tangent)

(A) 4 (B) 5 (C) 6 (D) 7 (E) the theorem does not apply

Typo on some worksheets!

and differentiable for $0<x<6$

8. If f is continuous for $0 \le x \le 6$ and $f(0)=1$ and $f(6)=7$, then which of the following could be false?



$m_{sec} = \frac{7-1}{6-0} = \frac{6}{6} = 1$

(A) f has a no vertical asymptotes on $0 \le x \le 6$. T, since f is continuous.

(B) There exists a value c on $0<c<6$ such that the slope of the tangent line at $x=c$ is 1. T, by MVT, $f'(c)=1$

(C) $f(c)=2$ for some c such that $0<c<6$. T, by IVT

(D) $f(c)=0$ for some c such that $0<c<6$. F, $y=0$ is not between $y=1$ & $y=7$

(E) $\lim_{x \to c} f(x)$ exists for all values c on $0<c<6$. ⇒ This really means f is continuous, so True.

ANSWERS:		
1) 2	4) A	7) B
2) $\pm\frac{\sqrt{3}}{3}$	5) C	8) D
3) E	6) D	