

AP CALCULUS BC
Stuff you MUST Know Cold

l'Hopital's Rule

If $\frac{f(a)}{g(a)} = \frac{0}{0}$ or $\frac{\infty}{\infty}$,
 then $\lim_{x \rightarrow a} \frac{f(x)}{g(x)} = \lim_{x \rightarrow a} \frac{f'(x)}{g'(x)}$

**Average Rate of Change
 (slope of the secant line)**

If the points $(a, f(a))$ and $(b, f(b))$ are on the graph of $f(x)$ the average rate of change of $f(x)$ on the interval $[a, b]$ is

$$\frac{f(b) - f(a)}{b - a}$$

**Definition of Derivative
 (slope of the tangent line)**

$$f'(x) = \lim_{h \rightarrow 0} \left(\frac{f(x+h) - f(x)}{h} \right)$$

Derivatives

$$\frac{d}{dx}(x^n) = nx^{n-1}$$

$$\frac{d}{dx}(\sin x) = \cos x$$

$$\frac{d}{dx}(\cos x) = -\sin x$$

$$\frac{d}{dx}(\tan x) = \sec^2 x$$

$$\frac{d}{dx}(\cot x) = -\csc^2 x$$

$$\frac{d}{dx}(\sec x) = \tan x \sec x$$

$$\frac{d}{dx}(\csc x) = -\cot x \csc x$$

$$\frac{d}{dx}(\ln u) = \frac{1}{u} du$$

$$\frac{d}{dx}(e^u) = e^u du$$

$$\frac{d}{dx}(\log_a x) = \frac{1}{x \ln a}$$

$$\frac{d}{dx}(a^u) = a^x (\ln a) du$$

Properties of Log and Ln

1. $\ln 1 = 0$
2. $\ln e^a = a$
3. $e^{\ln x} = x$
4. $\ln x^n = n \ln x$
5. $\ln(ab) = \ln a + \ln b$
6. $\ln\left(\frac{a}{b}\right) = \ln a - \ln b$

Differentiation Rules

Chain Rule

$$\frac{d}{dx}[f(u)] = f'(u) \frac{du}{dx}$$

Product Rule

$$\frac{d}{dx}(uv) = u \frac{dv}{dx} + v \frac{du}{dx}$$

Quotient Rule

$$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$$

Mean Value & Rolle's Theorem

If the function $f(x)$ is continuous on $[a, b]$ and the first derivative exists on the interval (a, b) , then there exists a number $x = c$ on (a, b) such that $f'(c) = \frac{f(b) - f(a)}{b - a}$
 if $f(a) = f(b)$, then $f'(c) = 0$.

Curve sketching and analysis

$y = f(x)$ must be continuous at each:

critical point: $\frac{dy}{dx} = 0$ or undefined.

local minimum: $\frac{dy}{dx}$ goes $(-, 0, +)$ or

$$(-, \text{und}, +) \text{ or } \frac{d^2 y}{dx^2} > 0$$

local maximum: $\frac{dy}{dx}$ goes $(+, 0, -)$ or

$$(+, \text{und}, -) \text{ or } \frac{d^2 y}{dx^2} < 0$$

Absolute Max/Min.: Compare local extreme values to values at endpoints.

pt of inflection : concavity changes.

$$\frac{d^2 y}{dx^2} \text{ goes } (+, 0, -), (-, 0, +), (+, \text{und}, -), \text{ or } (-, \text{und}, +)$$

"PLUS A CONSTANT"

The Fundamental Theorem of Calculus

$$\int_a^b f(x) dx = F(b) - F(a)$$

where $F'(x) = f(x)$

2nd Fundamental Theorem of Calculus

$$\frac{d}{dx} \int_a^{g(x)} f(x) dx = f(g(x)) \cdot g'(x)$$

Average Value

If the function $f(x)$ is continuous on $[a, b]$ and the first derivative exist on the interval (a, b) , then there exists a number $x = c$ on (a, b) such that

$$f(c) = \frac{1}{b-a} \int_a^b f(x) dx$$

$f(c)$ is the average value

Euler's Method

If given that $\frac{dy}{dx} = f(x, y)$ and

that the solution passes through (x_0, y_0) , then

$$x_{\text{new}} = x_{\text{old}} + \Delta x$$

$$y_{\text{new}} = y_{\text{old}} + \frac{dy}{dx}(x_{\text{old}}, y_{\text{old}}) \cdot \Delta x$$

Logistics Curves

$$P(t) = \frac{L}{1 + Ce^{-(Lk)t}}$$

where L is carrying capacity
 Maximum growth rate occurs when $P = \frac{1}{2} L$

$$\frac{dP}{dt} = kP(L - P) \text{ or}$$

$$\frac{dP}{dt} = (Lk)P\left(1 - \frac{P}{L}\right)$$

Integrals

$$\int kf(u)du = k \int f(u)du$$

$$\int du = u + C$$

$$\int u^n du = \frac{u^{n+1}}{n+1} + C, n \neq -1$$

$$\int \frac{1}{u} du = \ln |u| + C$$

$$\int e^u du = e^u + C$$

$$\int a^u du = \left(\frac{1}{\ln a} \right) a^u + C$$

$$\int \cos u du = \sin u + C$$

$$\int \sin u du = -\cos u + C$$

$$\int \tan u du = -\ln |\cos u| + C$$

$$\int \cot u du = \ln |\sin u| + C$$

$$\int \sec u du = \ln |\sec u + \tan u| + C$$

$$\int \csc u du = -\ln |\csc u + \cot u| + C$$

$$\int \frac{du}{u\sqrt{u^2 - a^2}} = \frac{1}{a} \operatorname{arcsec} \left(\frac{|u|}{a} \right) + C$$

$$\int \frac{du}{\sqrt{a^2 - u^2}} = \arcsin \left(\frac{u}{a} \right) + C$$

$$\int \frac{du}{a^2 + u^2} = \frac{1}{a} \arctan \left(\frac{u}{a} \right) + C$$

Integration by Parts

$$\int u dv = uv - \int v du$$

Arc Length

For a function, $f(x)$

$$L = \int_a^b \sqrt{1 + [f'(x)]^2} dx$$

For a polar graph, $r(\theta)$

$$L = \int_{\theta_1}^{\theta_2} \sqrt{[r(\theta)]^2 + [r'(\theta)]^2} d\theta$$

Lagrange Error Bound

If $P_n(x)$ is the n th degree Taylor polynomial of $f(x)$ about c , then

$$|f(x) - P_n(x)| \leq \frac{\max |f^{(n+1)}(z)|}{(n+1)!} |x - c|^{n+1}$$

for all z between x and c .

Distance, velocity and Acceleration

$$\text{Velocity} = \frac{d}{dt} (\text{position})$$

$$\text{Acceleration} = \frac{d}{dt} (\text{velocity})$$

$$\text{Velocity Vector} = \left\langle \frac{dx}{dt}, \frac{dy}{dt} \right\rangle$$

$$\text{Speed} = |v(t)| = \sqrt{(x')^2 + (y')^2}$$

Distance Traveled =

$$\int_{\text{initial time}}^{\text{final time}} |v(t)| dt = \int_{\text{initial time}}^{\text{final time}} \sqrt{(x')^2 + (y')^2} dt$$

$$x(b) = x(a) + \int_a^b x'(t) dt$$

$$y(b) = y(a) + \int_a^b y'(t) dt$$

Polar Curves

For a polar curve $r(\theta)$, the

$$\text{Area inside a "leaf" is } \frac{1}{2} \int_{\theta_1}^{\theta_2} [r(\theta)]^2 d\theta$$

where θ_1 and θ_2 are the "first" two times that $r = 0$.

The slope of $r(\theta)$ at a given θ is

$$\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta} = \frac{\frac{d}{d\theta} [r(\theta) \sin \theta]}{\frac{d}{d\theta} [r(\theta) \cos \theta]}$$

Ratio Test

(use for interval of convergence)

The series $\sum_{n=0}^{\infty} a_n$ converges if

$$\lim_{n \rightarrow \infty} \left| \frac{a_{n+1}}{a_n} \right| < 1 \quad \text{CHECK ENDPOINTS}$$

Alternating Series Error Bound

If $S_N = \sum_{n=1}^N (-1)^n a_n$ is the N^{th} partial sum of a convergent alternating series, then

$$|S_{\infty} - S_N| \leq |a_{N+1}|$$

Most Common Series

$$\sum_{n=1}^{\infty} \frac{1}{n} \text{ diverges} \quad \sum_{n=1}^{\infty} \frac{(-1)^n}{n} \text{ converges} \quad \sum_{n=0}^{\infty} A(r)^n \text{ converges to } \frac{A}{1-r} \text{ if } |r| < 1$$

Volume

Solids of Revolution

$$\text{Disk Method: } V = \pi \int_a^b [R(x)]^2 dx$$

Washer Method:

$$V = \pi \int_a^b ([R(x)]^2 - [r(x)]^2) dx$$

$$\text{Shell Method: } V = 2\pi \int_a^b r(x)h(x)dx$$

Volume of Known Cross Sections

Perpendicular to

x-axis:

$$V = \int_a^b A(x) dx$$

y-axis:

$$V = \int_c^d A(y) dy$$

Taylor Series

If the function f is "smooth" at $x = c$, then it can be approximated by the n th degree polynomial

$$f(x) \approx f(c) + f'(c)(x - c)$$

$$\begin{aligned} &+ \frac{f''(c)}{2!} (x - c)^2 + \dots \\ &+ \frac{f'''(c)}{3!} (x - c)^3 + \dots \\ &+ \frac{f^{(n)}(c)}{n!} (x - c)^n \end{aligned}$$

Elementary Functions

Centered at $x = 0$

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \dots$$

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \dots$$

$$\frac{1}{1-x} = 1 + x + x^2 + x^3 + \dots$$

$$\ln(x+1) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \dots$$